

# Scaling of elliptic flow in heavy-ion collisions with the number of constituent quarks in a transport model

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We studied the number of constituent quark scaling (NCQ) behaviour of elliptic flow ( $v_2$ ) under the framework of A Multi-Phase Transport model (AMPT) at both top-RHIC and LHC energies. The NCQ-scaling in  $v_2$  holds at top-RHIC energy with AMPT string melting version, while it breaks in Pb+Pb collisions at LHC energy using the same framework. The breaking of NCQ-scaling at LHC energy has been studied by varying the magnitude of parton-parton scattering cross-section and lifetime of hadronic cascade as implemented in AMPT. We find that the breaking of NCQ scaling in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV is independent of the magnitude of parton-parton cross-section and the later stage hadronic interactions. Further we observed that scaling holds in a small collision system like Si+Si at  $\sqrt{s_{NN}} = 2.76$  TeV. We discussed that the breaking of NCQ scaling is possibly due to high phase-space density of constituents quarks in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV.

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## I. INTRODUCTION

Relativistic heavy-ion collision experiments aim to study the formation and evolution of a strongly interacting matter called Quark Gluon Plasma (QGP) [1]. Experiments at Brookhaven Relativistic Heavy Ion Collider (RHIC) and at CERN Large Hadron Collider (LHC) established the existence of such strongly interacting matter, which is expected to be formed micro-seconds after the big-bang.

The elliptic flow parameter,  $v_2$ , which is defined as a second harmonic coefficient of the azimuthal Fourier decomposition of the momentum distribution of produced particles has been widely used as an excellent tool for understanding the dynamics of the system formed in the early stages of high-energy heavy-ion collisions [2–8]. This flow parameter  $v_2$  is extracted by studying the correlation of produced particles with respect to the reaction plane ( $\Psi$ ) as,

$$v_2 = \langle \cos(2(\phi - \Psi)) \rangle, \quad (1)$$

where  $\phi$  is the azimuthal angle of the produced particles [9].

Results from RHIC-experiments show that at low transverse momentum ( $p_T < 2$  GeV/c), there is a clear mass-ordering of  $v_2$  among the identified hadrons [10, 11]. It is observed that at fixed  $p_T$ , heavier hadrons have smaller values of  $v_2$  than the lighter ones. Hydrodynamic calculations suggest that the interplay between radial and elliptic flow plays an important role in determining the mass-ordering of  $v_2$  at low  $p_T$  [2–7]. Subsequent later stage hadronic re-scattering can also distort  $v_2$  at low  $p_T$  [12]. It is observed that in the intermediate- $p_T$  region ( $2.0 < p_T < 4.0$  GeV/c), the  $p_T$ -differential  $v_2$  of baryons and mesons form separate groups [10, 11].

Such a baryon-meson splitting in  $v_2$  is successfully reproduced by models where quark-coalescence mechanism is considered to be the dominant process for hadronization in this  $p_T$ -regime [13, 14]. When both  $v_2$  and  $p_T$  of identified hadrons are divided by number of constituent quarks ( $n_q$ ), all the hadrons follow an approximate scaling behaviour. This is known as number of constituent quark (NCQ) scaling. The origin of such scaling is interpreted as an evidence for dominance of quark degrees of freedom in the early stages of heavy-ion collision. Another way of representing NCQ scaling is to plot  $n_q$  scaled  $v_2$  as a function of  $(m_T - m_0)/n_q$ , where  $m_T$  is transverse mass and  $m_0$  is the rest mass of hadron.

Recent  $v_2$  results from LHC [15] show similar trend of mass-ordering among the identified hadrons at low  $p_T$  ( $< 3$  GeV/c) and about 30% increase in radial flow than the top-RHIC energy. But in the intermediate  $p_T$  region ( $3.0 < p_T < 6.0$  GeV/c), the  $v_2$  results do not seem to follow NCQ-scaling as observed in lower energy RHIC experiments. The  $v_2$  of identified hadrons at LHC energy deviates from NCQ-scaling at a level of 20%. This observation has triggered theoretical debate over the NCQ-scaling.

A Multi-Phase Transport (AMPT) model with string melting version (which includes parton coalescence) has been used to reproduce the observed NCQ-scaling in  $v_2$  at top-RHIC energies [16]. In this paper, we investigated the behaviour of NCQ-scaling both at top-RHIC and LHC energies using the framework of AMPT model to understand the reason behind it's breaking at higher energies.

This paper is organized as follows. In section II, we briefly discuss the AMPT model. In section III, we describe the NCQ-scaling behaviour of  $v_2$  of identified hadrons at top-RHIC and LHC energies using

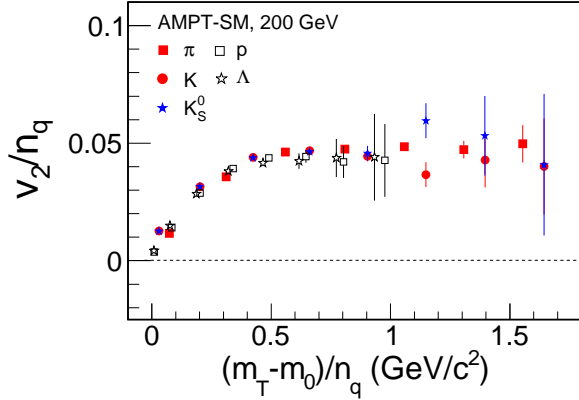


FIG. 1: (Color online)  $v_2/n_q$  as a function of  $(m_T - m_0)/n_q$  for some selected hadrons ( $\pi$ ,  $K$ ,  $K_S^0$ ,  $p$  and  $\Lambda$ ) in minimum bias Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV using AMPT-SM model. The parton-parton cross-section is taken as 3 mb with hadronic cascade time = 30 fm in AMPT-SM model.

the AMPT model (version 1.11). The results are summarized in section IV.

## II. THE AMPT MODEL

The AMPT model, which is a hybrid transport model, has four main stages: the initial conditions, partonic interactions, the conversion from the partonic to the hadronic matter, and hadronic interactions [16]. It uses the same initial conditions as HIJING [17]. Scattering among partons are modelled by Zhang's parton cascade [18], which calculates two-body parton scatterings using cross sections from pQCD with screening masses. In the default AMPT model, partons are recombined with their parent strings and when they stop interacting, the resulting strings fragment into hadrons according to the Lund string fragmentation model [19]. However, in the string melting scenario (labeled as AMPT-SM), these strings are converted to soft partons and a quark coalescence model is used to combine partons into hadrons. The evolution dynamics of the hadronic matter is described by A Relativistic Transport (ART) model. The interactions between the minijet partons in the AMPT Default model and those between partons in the AMPT-SM could give rise to substantial  $v_2$ . The parton-parton interaction cross section in the string-melting version of the AMPT is taken to be 3 mb and 10 mb. In this study, approximately 500 K (50 K) events for each configuration were generated for minimum-bias Au+Au (Pb+Pb) collisions.

## III. RESULTS AND DISCUSSION

It has been observed that NCQ scaling in  $v_2$  holds for AMPT with string melting scenario, which incorporates partonic coalescence mechanism, but no such scaling occurs in the default AMPT [20]. We studied the energy dependence of such scaling using AMPT-SM, mainly at top RHIC and LHC energies. Fig 1 shows  $v_2/n_q$  as a function of  $(m_T - m_0)/n_q$  for some selected hadrons ( $\pi$ ,  $K$ ,  $K_S^0$ ,  $p$  and  $\Lambda$ ) in minimum bias Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV using AMPT-SM model. A clear scaling is observed among all hadrons consistent with the observation in Ref [20]. Here we used parton-parton cross-section ( $\sigma_{PP}$ ) equal to 3 mb and hadron cascade time ( $\tau$ ) equal to 30 fm in these results.

After observing a clear scaling at 200 GeV, we studied NCQ scaling in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV using AMPT-SM model as shown in Fig. 2. A clear breaking of scaling is observed for  $(m_T - m_0)/n_q > 0.4$  GeV/ $c^2$ , which is very striking and interesting as we have used AMPT-SM model. Fig. 2(a) and 2(b) show scaling results where the magnitude of  $\sigma_{PP}$  has been taken as 3 mb and 10 mb, respectively, keeping same hadron cascade time (30 fm). It is clear that scaling breaks down for both the values of  $\sigma_{PP}$ . This indicates that the breakdown of NCQ scaling at  $\sqrt{s_{NN}} = 2.76$  TeV is independent of magnitude of parton-parton cross-section.

One possible reason for the violation may be the distortion of initially developed  $v_2$  by later hadronic interaction. To check this effect, we turn-off hadronic cascade in AMPT model. This can be done by setting input parameter  $Nt=3$ , which gives hadron cascade time equal to 0.6 fm (minimum hadron cascade time in AMPT). The NCQ scaling result from AMPT-SM ( $\sigma_{PP}= 10$  mb) with hadron cascade time 0.6 fm is shown in Fig. 3. In this case too we have observed that the scaling breaks, indicating that it is not due to the hadronic interactions at  $\sqrt{s_{NN}} = 2.76$  TeV.

### A. Quark- $v_2(p_T)$ distributions in AMPT model

According to coalescence model the relation between quark- $v_2$  ( $v_2^q$ ) and hadrons- $v_2$  ( $v_2^h$ ) is as follows:

$$v_2^h(p_T) = n_q v_2^q(p_T/n_q). \quad (2)$$

Where  $p_T$  is the transverse momentum of hadron. The violation of NCQ scaling at LHC energy within a parton coalescence approach was first predicted in Ref [21]. According to Ref [21], modifications of the underlying light and heavy quark  $v_2(p_T)$  due to the

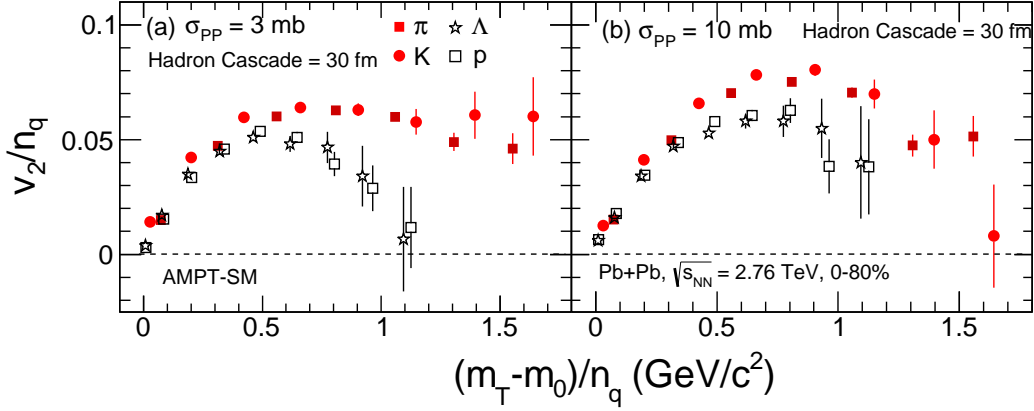


FIG. 2: (Color online)  $v_2/n_q$  as a function of  $(m_T - m_0)/n_q$  for some selected hadrons ( $\pi$ ,  $K$ ,  $p$  and  $\Lambda$ ) in minimum bias (a) Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV (b) Pb+Pb collisions at 2.76 TeV using AMPT-SM model.

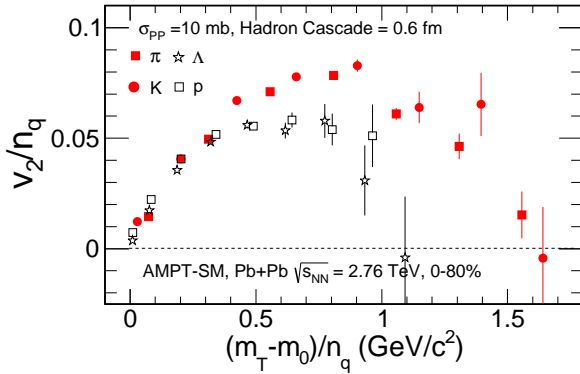


FIG. 3: (Color online)  $v_2/n_q$  as a function of  $(m_T - m_0)/n_q$  for some selected hadrons ( $\pi$ ,  $K$ ,  $p$  and  $\Lambda$ ) in minimum bias Pb+Pb collisions at 2.76 TeV using AMPT-SM model ( $\sigma_{PP} = 10$  mb,  $\tau = 0.6$  fm).

strong transverse expansion at LHC energy could be the reason for NCQ scaling violation. To understand such behavior in Pb+Pb collisions at 2.76 TeV, we have checked underlying  $v_2(p_T)$  for different quark flavors in AMPT-SM model.

The  $v_2$  of  $u$ ,  $d$  and  $s$  quarks as a function of  $p_T$  in the AMPT-SM model ( $\sigma_{PP} = 3$  mb) are shown in Fig. 4 (a) and 4 (b) for  $\sqrt{s_{NN}} = 200$  GeV (Au+Au) and 2.76 TeV (Pb+Pb), respectively. Ratios with respect to  $u$ -quark  $v_2$  are shown in the corresponding lower panel. We have observed that the  $v_2(p_T)$  of  $u$ ,  $d$  and  $s$  quarks are the same for both the energies in AMPT-SM model. However, for  $p_T < 0.5$  GeV/c, magnitude of  $s$ -quark  $v_2$  is slightly lower than that of  $u$  and  $d$ . It is clear from Fig. 4 that the  $v_2(p_T)$  distribution for different quark flavors is similar for both Au+Au and Pb+Pb collisions at  $\sqrt{s_{NN}} = 200$  GeV

and 2.76 TeV, respectively. Therefore, the breaking of NCQ scaling in AMPT-SM model for Pb+Pb collision at  $\sqrt{s_{NN}} = 2.76$  TeV is not due to change in  $v_2(p_T)$  of underlying quarks.

### B. Effect of parton density in coalescence mechanism

Let us recall the formalism of coalescence mechanism. In a simplified coalescence scenario, the probability that the constituents  $a$  and  $b$  will form a composite object  $C$  [13] is

$$f_C(PC, R, t_c) \approx f_a(m_a PC / (m_a + m_b), R, t_c) \times f_b(m_b PC / (m_a + m_b), R, t_c). \quad (3)$$

Here  $f_i$  denotes phase densities,  $PC$  is the momentum of the composite particle,  $t_c$  is the coalescence time and  $R$  is the centre-of-mass. Masses of constituents are denoted by  $m_i$ . Within the regime of coalescence mechanism, the invariant spectrum of produced particles is proportional to the product of the invariant spectra of constituents. Therefore, the yields of mesons and baryons produced by coalescence of quarks ( $q$ ) are given by

$$\frac{dN_B}{d^2p_T}(p_T) = f_B(p_T) \left[ \frac{dN_q}{d^2p_T}(p_T/3) \right]^3 \quad (4)$$

$$\frac{dN_M}{d^2p_T}(p_T) = f_M(p_T) \left[ \frac{dN_q}{d^2p_T}(p_T/2) \right]^2, \quad (5)$$

where the coefficient  $f_M$  and  $f_B$  are the probabilities for meson and baryon coalescence. Note that Eq. 2, 4 and 5, are valid only when the phase space density is very small [13]. When phase-space density

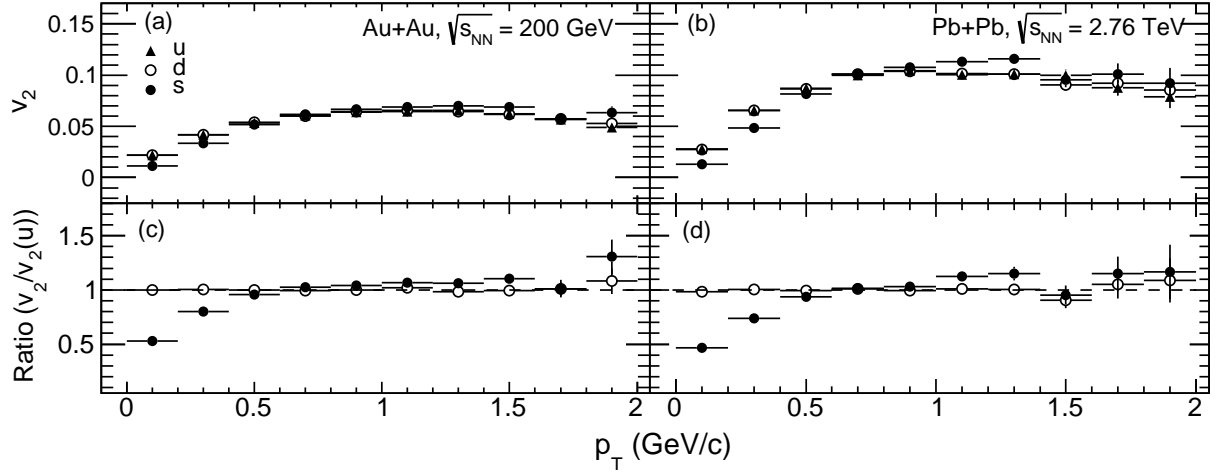


FIG. 4: (Color online) The  $v_2$  of  $u$ ,  $d$  and  $s$  quarks as a function of  $p_T$  in AMPT-SM model ( $\sigma_{PP} = 3$  mb) for minimum bias (a) Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV and (b) Pb+Pb collisions at 2.76 TeV. Ratios with respect to  $u$ -quark  $v_2$  are shown in corresponding lower panel.

of quarks is very high, the probability to find another quark in vicinity will be close to unity. So the final composite  $v_2$  of hadron will be linear in terms of the quark's  $v_2$  and hence breaking the scaling relation. On the other hand for low density, a quark has a small probability of finding another quarks to coalesce, and Eq. 2, 4 and 5 will be valid.

So the change in phase space density of quarks can affect coalescence mechanism and it can be studied using AMPT model. We generated 2 million Si+Si collision events at  $\sqrt{s_{NN}} = 2.76$  TeV using the same AMPT-SM configuration ( $\sigma_{PP} = 3$  mb,  $\tau = 0.6$  fm). Because of small system size, we would expect a smaller density compared to that in Pb+Pb collisions. So if NCQ-scaling at LHC energies in Pb+Pb collisions breaks due to high density of partons, the scaling might hold in Si+Si collision system at same centre-of-mass energy. Fig. 5 shows the NCQ scaling plot for minimum bias Si+Si system at  $\sqrt{s_{NN}} = 2.76$  TeV. We can see that NCQ scaling holds much better than Pb+Pb system. This confirms that the breaking of NCQ-scaling of  $v_2$  in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV is due to very high phase-space density of initially produced quarks.

#### IV. SUMMARY AND CONCLUSION

In summary, we have studied the number of constituent quark scaling in  $v_2$  for hadrons at top-RHIC and LHC energies using AMPT-SM model. We have observed that while NCQ-scaling holds at  $\sqrt{s_{NN}} = 200$  GeV but model fails to reproduce the same in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV. We have

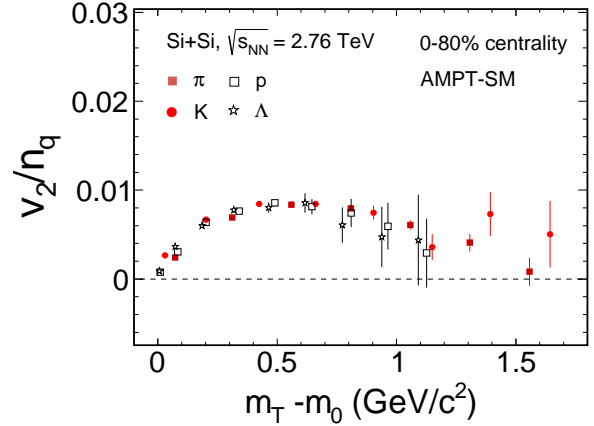


FIG. 5: (Color online)  $v_2/n_q$  as a function of  $(m_T - m_0)/n_q$  for some selected hadrons ( $\pi$ ,  $K$ ,  $p$  and  $\Lambda$ ) in minimum bias Si+Si collisions at  $\sqrt{s_{NN}} = 2.76$  TeV using AMPT-SM model ( $\sigma_{PP} = 3$  mb,  $\tau = 0.6$  fm).

observed the breaking in NCQ scaling at  $\sqrt{s_{NN}} = 2.76$  TeV is independent of the magnitude of parton-parton cross-section and also not due to later stage hadronic interactions. We also compared  $v_2$  of  $u$ ,  $d$  and  $s$  quarks as a function of  $p_T$  for Au+Au and Pb+Pb collisions in AMPT-SM model to see any possible change in underlying quark  $v_2(p_T)$  due large radial flow at LHC energy. We find  $v_2(p_T)$  of  $u$ ,  $d$  and  $s$  quarks shows similar behaviour for both Au+Au and Pb+Pb collisions. Therefore, the violation in NCQ scaling is not due to change in underlying quark  $v_2(p_T)$  in Pb+Pb collisions at LHC energy.

Further we checked the effect of parton's phase-space density on NCQ scaling behaviour within the framework of coalescence. We observed that the scaling holds in a small collision system like Si+Si at  $\sqrt{s_{NN}} = 2.76$  TeV where the phase-space density of constituent quarks is not very high as compared to Pb+Pb. This observation can be well understood in the framework of coalescence mechanism. Our

study shows that the NCQ-scaling in  $v_2$  is not a necessary condition for quark coalescence when phase-space density of constituent quarks is very high, e.g Pb+Pb collision LHC energies.

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